coefficient to be the principal one in the expansion of at least one of the functions. As shown by expansions (A.2) and (A.3) this is not so for $3\varkappa / 4 N_{Pr}$ outside that range.

In concluding the author thanks O.S. Ryzhov for his advice and interest.

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ON A CLASS OF SOLUTIONS OF THE NONLINEAR EQUATION FOR THE VELOCITY POTENTIAL

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We construct a class of exact solutions of the equation for the velocity potential of unsteady plane flows of a polytropic gas. These solutions contain an infinite number of arbitrary functions of a single argument. They are given in the form of series in rational powers of the characteristic argument in the space of variables of the time-velocity hodograph. We study the applications of the series obtained to solving certain problems of flows arising during the motion of curvilinear pistons through a gas, so that at the initial instant the normal velocity and

acceleration of the pistons are equal to zero. When the motion has a cylindrical symmetry, we study the convergence of the series obtained, over a short period of time. Numerical results obtained are quoted. The present paper is the continuation of the investigation begun in [1, 2].

1. Following [1] we write the equation for the function $\Psi\ (r,\ \varphi,\ t)$ which is an analog of the velocity potential $\Phi\ (x_1,\ x_2,\ t)$, in the following form

$$\begin{aligned}
&\Psi_{tt}\,(-r^{-2}\Psi_{r\varphi} - r^{-4}\Psi_{\varphi}{}^{2} + r^{-2}\Psi_{\varphi\varphi}\Psi_{rr} + r^{-1}\Psi_{r}\Psi_{rr} + \\
&2r^{-3}\Psi_{r\varphi}\Psi_{\varphi}) + \frac{1}{\varkappa}\left(\Psi_{t} - \frac{1}{2}r^{2}\right)(\Psi_{rr} + r^{-1}\Psi_{r} + r^{-2}\Psi_{\varphi\varphi}) - \\
&r^{-2}\Psi_{rt}{}^{2}\Psi_{\varphi\varphi} - r^{-1}\Psi_{rt}\Psi_{r} + 2r^{-2}\Psi_{rt}\Psi_{\varphi t}\Psi_{r\varphi} - 2r^{-3}\Psi_{\varphi t}\Psi_{\varphi}\Psi_{rt} - \\
&r^{-2}\Psi_{\varphi t}^{2}\Psi_{rr} + 2\,(r^{-1}\Psi_{rt}\Psi_{\varphi\varphi} + \Psi_{rt}\Psi_{r} - r^{-1}\Psi_{\varphi t}\Psi_{r\varphi} + r^{-2}\Psi_{\varphi t}\Psi_{\varphi}) - \\
&\Psi_{\varphi\varphi}^{2} - r\Psi_{r} = 0
\end{aligned} \qquad (1.1)$$

The subscripts accompanying Ψ denote differentiation with respect to the corresponding arguments, t is time and $r,\ \varphi$ are the polar coordinates in the velocity $u_1,\ u_2$ hodograph plane. The function Ψ is related to the potential Φ by the following formula

$$\Psi = x_1u_1 + x_2u_2 - \Phi + Mt$$

$$(\Phi_{x_1} = u_1 = r\cos\varphi,\ \ \Phi_{x_2} = u_2 = r\sin\varphi)$$

where M is a constant appearing in the Cauchy integral (c denotes the speed of sound and γ is the adiabatic index)

$$c^2 = \frac{1}{\varkappa}\left(M - \Phi_t - \frac{1}{2}\sum_i \Phi_{x_i}^2\right), \qquad \varkappa = \frac{1}{\gamma - 1}$$

Having determined the function Ψ we can obtain the flow in the physical space x_1, x_2, t from the formulas

$$x_1 = \Psi_r\cos\varphi - r^{-1}\,\Psi_\varphi\sin\varphi, \quad x_2 = \Psi_r\sin\varphi + r^{-1}\,\Psi_\varphi\cos\varphi \qquad (1.2)$$

In [2, 3] the authors constructed a class of solutions for (1.1) in the form

$$\Psi\,(r,\ \varphi,\ t) = \sum_{k=0}^{\infty} a^{(k)}\,(\varphi,\ t)\,r^{(k)}$$

where $a^{(k)}$ were determined from certain linear differential equations and contained arbitrary functions of φ. This class of solutions was then used to solve the following problem.

Let at the initial instant $t = 0$ a homogeneous polytropic gas in which the speed of sound $c = 1$ be at rest inside or outside a reasonably smooth closed convex cylindrical surface S_0. At instant $t = 0$ piston S_t begins to move at zero initial normal velocity V_n and with nonzero initial normal acceleration $W_n \neq 0$.

The problem was to find the solution of the nonlinear equation of velocity potential $\Phi\ (x_1,\ x_2,\ t)$ in the region bounded by the surface of piston S_t and of the weak discontinuity L_t which at the initial instant $t = 0$ separates from surface S_0 and propagates through the quiescent gas at unit normal velocity.

We shall consider the following problem, more general than the one solved in [2].

Let the motion of the piston surface S_t be defined by the equations

$$x_1 = x_1(\beta, t), \qquad x_2 = x_2(\beta, t), \qquad \beta \in [0, b] \tag{1.3}$$

where β is a parameter such that $\beta = \varphi$ when $t = 0$, and let the equation of the piston surface S_0 be determined by

$$x_1 = f(\varphi)\cos\varphi - f'(\varphi)\sin\varphi, \qquad x_2 = f(\varphi)\sin\varphi + f'(\varphi)\cos\varphi$$

The function $f(\varphi)$ defines the arbitrariness in the choice of the form of S_0; the surface of weak discontinuity L_0 coincides with it when $t = 0$. We shall assume that the normal velocity of the piston computed by the formula

$$V_n(t, \beta) = \frac{D(x_1, x_2)}{D(\beta, t)}\left[\left(\frac{\partial x_1}{\partial t}\right)^2 + \left(\frac{\partial x_2}{\partial t}\right)^2\right]^{1/2}$$

can be expanded at small values of t into a convergent series of the form

$$V_n(t, \beta) = \sum_{k=n}^{\infty} g^{(k)}(\beta)\, t^{(k)}, \qquad n \geqslant 1 \tag{1.4}$$

We seek a solution in the region of perturbrd motion of gas between the surfaces S_t and L_t (the case $n = 1$ was analyzed in [2]). Let us construct the solution of the problem formulated above in the form

$$\Psi(r, \varphi, t) = a_0 + a_1(\varphi, t)r + \sum_{k=0}^{\infty} b^{(k+1)}(\varphi, t)\, r^{(k+n+1)/n} \tag{1.5}$$

From the results of [2] it follows that

$$a_0 = \varkappa t + \text{const}, \qquad a_1 = t + f(\varphi) \tag{1.6}$$

Substituting the series (1.5) into (1.1) and equating to zero the coefficients of $r^{(k+n+1)/n}$ we obtain a system of linear differential equations from which the coefficients $b^{(k+1)}$ $(k \geqslant 0)$ are found one after another.

Since the formulas become very cumbersome, in the following we shall limit our discussions to the case $n = 2$ (this corresponds to the case when at the initial instant $t = 0$ the normal accelerations and velocity of the piston are zero, but the derivative of the normal acceleration is not zero). The system defining the coeffcients $b^{(k)}$ $(n = 2)$ has the form

$$-3(t + f + f'')b_t^{(1)} + {}^3/_4\, b^{(1)} = 0 \tag{1.7}$$

$$\begin{aligned}&-4(t + f + f'')b_t^{(2)} + 2b^{(2)} + (t + f + f'')(\varkappa^{-1} + 2) + \\ &\quad b_{tt}^{(1)}b_\varphi^{(1)}[2t + 2f - f'] + {}^3/_4 b_{tt}^{(1)}b^{(1)}[t + f + 2f''] + \\ &\quad b_{\varphi t}^{(1)}b_\varphi^{(1)}[3\varkappa^{-1} - 2] - 3b_{\varphi t}^{(1)}b_t^{(1)}f' - {}^9/_4(b_t^{(1)})^2[t + f + f''] - \\ &\quad {}^9/_2 b_t^{(1)}b^{(1)} - 3b_{\varphi\varphi}^{(1)}b_t^{(1)} = 0\end{aligned}$$

$$\begin{aligned}&-(k + 3)(t + f + f'')b_t^{(k+1)} + {}^1/_4(k + 3)(k + 1)b^{(k+1)} + \\ &\quad {}^1/_8 F^{(k+1)}(\varphi, t) = 0 \qquad (k \geqslant 2)\end{aligned}$$

the functions $F^{(k+1)}(\varphi, t)$ depend on the functions $a_0, a_1, b^{(m)}(\varphi, t)$ $(m < k + 1)$ and on their first and second order derivatives, and can be written out in the explicit form. Let us write the expressions for $F^{(k+1)}(t)$ for the case of a motion with cylindrical symmetry (the expressions for $F^{(k+1)}(\varphi, t)$ become very cumbersome in the general case) (*)

$$F^{(k+1)}(t) = \sum_{m=0}^{k-1} \{2(m+3)(m+2)a_1 b_{tt}^{(k-m)} b^{(m+1)} - \tag{1.8}$$
$$2(k+2-m)a_1 b^{(k-m)} b_{tt}^{(m+1)} - 4(m+3)(k+2-m) b^{(k-m)} b_t^{(m+1)} -$$
$$2(m+3)(k+2-m)a_1 b_t^{(m+1)} b_t^{(k-m)}\} +$$
$$\sum_{p,\,m=0}^{k-2} \{(3+p-m)(k+1-p)(k-p) b_{tt}^{(m+1)} b^{(p+1-m)} b^{(k-1-p)} -$$
$$(m+3)(p+3-m) b_{tt}^{(k-1-p)} b^{(p+1-m)} b^{(m+1)} +$$
$$2(k+1)\left(\frac{2k+1}{\varkappa}+4\right) b^{(k-1)} + 2a_1\left(\frac{3}{\varkappa}+4(k+1)\right) b_t^{(k-1)} +$$
$$(m+3)(p+3-m)(k+1-p) b_t^{(p+1-m)} b_t^{(m+1)} b^{(k-1-p)}\} +$$
$$\sum_{m=0}^{k-3} \left\{\frac{2}{\varkappa}(k-m)^2 b_t^{(m+1)} b^{(k-2-m)} + 4(m+3)(k-m) b_t^{(m+1)} b^{(k-2-m)}\right\}$$
$$-(k-1)\left(\frac{1}{\varkappa}+4\right) b^{(k-3)} + 8\left(\frac{1}{\varkappa}+2\right) c^{(k-1)} - \frac{2}{\varkappa} c^{(k-2)} - 8c^{(k-3)}$$
$$b^{(-k)} \equiv 0, \quad k \geq 0, \quad c^{(k)} = \begin{cases} a_1 & k = 0 \\ 0, & k \neq 0 \end{cases}$$

The general solution of (1.7) has the form

$$b^{(k+1)} = (t+f+f'')^{(k+1)/4} \left[c^{(k+1)}(\varphi) + \frac{1}{8(k+3)} \int F^{(k+1)}(\varphi, t)(t+f+f'')^{-(k+5)/4} dt\right] \tag{1.9}$$

where $c^{(k+1)}(\varphi)$ are arbitrary functions which can be determined from the law of motion of the piston S_t.

2. Let us determine the functions $c^{(k+1)}(\varphi)$. Following [2] we write the kinematic condition of motion

$$r_*(\beta, t)(\cos\varphi_*(\beta, t) n_1(\beta, t) + \sin\varphi_*(\beta, t) n_2(\beta, t)) = V_n(\beta, t) \tag{2.1}$$

Here $r_*(\beta, t)$ and $\varphi_*(\beta, t)$ are certain, a priori unknown functions such that $u_1{}^* = r_* \cos\varphi_*$, $u_2{}^* = r_* \sin\varphi_*$ define the components of the velocity vector at the piston ($\varphi_*(\varphi, 0) = \varphi$, $r_*(\varphi, 0) = 0$). The functions $r_*(\beta, t)$ and $\varphi_*(\beta, t)$ are obtained from the system of equations

$$x_1(\beta, t) = \cos\varphi_* \left[t + f(\varphi_*) + \frac{1}{2}\sum_{k=0}^{\infty}(k+3) b^{(k+1)}(\varphi_*, t) r_*^{(k+1)/2}\right] - \tag{2.2}$$
$$\sin\varphi_* \left[f'(\varphi_*) + \sum_{k=0}^{\infty} b_\varphi^{(k+1)}(\varphi_*, t) r_*^{(k+1)/2}\right]$$
$$x_2(\beta, t) = \sin\varphi_* \left[t + f(\varphi_*) + \frac{1}{2}\sum_{k=0}^{\infty}(k+3) b^{(k+1)}(\varphi_*, t) r_*^{(k+1)/2}\right] +$$
$$\cos\varphi_* \left[f'(\varphi_*) + \sum_{k=0}^{\infty} b_\varphi^{(k+1)}(\varphi_*, t) r_*^{(k+1)/2}\right]$$

---

*) The detailed computations and expressions for these functions can be found in a report deposited in the library of the Mathematics and Mechanics Institute of the Ural Research Center of the Academy of Sciences U.S.S.R.

The algorithm used to determine the arbitrary $c^{(k+1)}(\varphi)$ is basically analogous to that given in [2], but differs from it in the fact that the differentiation of (2.2) with respect to t gives rise to indeterminacies of the form $r_*^{-n-1/2}\partial^{n+1}r_* / \partial t^{n+1}$ ($r_* \to 0$ when $t \to 0$) and these must be expanded using (1.4).

The relation (2.1) is an identity with respect to the variables β and t. Let us differentiate (2.1) with respect to t and assume $t = 0$. We also differentiate (2.2) with respect to t and pass to the limit as $t \to 0$. The resulting sequence of equations enables us to compute all $c^{(k+1)}(\varphi)$ $(k \geqslant 0)$. Thus, after the first differentiation of (2.1) and (2.2) we have the equation

$$\frac{\partial x_1(0,\varphi)}{\partial t}\cos\varphi + \frac{\partial x_2(0,\varphi)}{\partial t}\sin\varphi = V_n(0,\varphi) = \tag{2.3}$$

$$1 + \lim_{t\to 0}\frac{3}{4}r_*^{-1/2}\frac{\partial r_*}{\partial t}(\varphi_*, t)\, b^{(1)} = 0$$

for definition of $c^{(1)}(\varphi)$. From (1.4), (2.3) and (1.9) we obtain

$$\lim_{t\to 0}\frac{3}{4}r_*^{-1/2}\frac{\partial r_*(\varphi_*, t)}{\partial t}b^{(1)} = \frac{3}{2}\sqrt{g^{(2)}(\varphi)}\,b^{(1)}(\varphi)$$

$$c^{(1)}(\varphi) = -\frac{2}{3\sqrt{g^{(2)}(\varphi)}\,(f+f'')^{1/4}}$$

In the next step we find $c^{(2)}(\varphi)$ by differentiating (2.2) twice with respect to t and constructing linear combinations, we obtain the following equations:

$$\frac{\partial^2 x_1(0,\varphi)}{\partial t^2}\cos\varphi + \frac{\partial^2 x_2(0,\varphi)}{\partial t^2}\sin\varphi = 2b^{(2)}(0,\varphi)\frac{\partial^2 r_*(0,\varphi)}{\partial t^2} -$$

$$\frac{\partial\varphi_*(0,\varphi)}{\partial t}[f(\varphi) + f''(\varphi)] + \lim_{t\to 0} r_*^{-1/2}\frac{\partial r_*}{\partial t}\left[\frac{3}{2}b_t^{(1)} + b^{(1)}\frac{\partial\varphi_*}{\partial t}\right]$$

where $\partial\varphi_*(0,\varphi) / \partial t$ and $\partial^2 r_*(0,\varphi) / \partial t^2$ are defined by formulas

$$\frac{\partial\varphi_*(0,t)}{\partial t} = (f(\varphi) + f''(\varphi))^{-1}\left(\frac{\partial\Gamma}{\partial t} - \lim_{t\to 0} r_*^{-1/2}\frac{\partial r_*}{\partial t}b^{(1)}\right)$$

$$\Gamma = -x_1(0,\varphi)\sin\varphi + x_2(0,\varphi)\cos\varphi, \qquad \frac{\partial^2 r_*(0,\varphi)}{\partial t} = \frac{\partial^2 V_n(0,\varphi)}{\partial t^2}$$

It is evident that the above procedure can be continued. The condition $f(\varphi) + f''(\varphi) \neq 0$ ensures that the functions $c^{(k+1)}(\varphi)$ $(k \geqslant 0)$ can be determined uniquely. The equation for the determination of the other constants $C^{(k+1)}(\varphi)$ $(k \geqslant 2)$ is very cumbersome and shall not be given here.

3. Let us consider in more detail the case of motions with cylindrical symmetry when the piston S_t represents, at $t = 0$, a cylinder of radius R_0. Then the formulas (1.2) will be replaced by

$$\xi = \sqrt{x_1^2 + x_2^2} = \Psi_r(r, t) \tag{3.1}$$

Consider the problem of disruption of perturbed potential compression flows generated by the motion of the cylinder S_t. We compute some of the coefficients of the series (1.5)

$$b^{(1)} = -\frac{2R_0^{-1/4}(t+R_0)^{1/4}}{3G}$$

$$b^{(2)} = -\left[\frac{R_0^{-3/2}}{32G^2} + \frac{(\gamma+1)R_0^{1/2}}{2}\right](t+R_0)^{1/2} + \frac{(\gamma+1)(t+R_0)}{2} + \frac{5R_0^{-1/2}(t+R_0)^{-1/2}}{32G^2}$$

$$b^{(3)} = \left[\frac{3137R_0^{-11/4}}{23040G^3} + \left(\frac{\ln R_0}{G^3} - \frac{5}{64G^3}\right)R_0^{-7/4} + \frac{g^{(3)}R_0^{-5/4}}{64G^5} + \right.$$
$$\left(\frac{11\gamma+29}{48G} + \frac{1}{18G^7}\left(\frac{9}{2}G^2g^{(4)} - \frac{9}{8}[g^{(3)}]^2\right)\right)R_0^{-3/4} +$$
$$\left(\frac{(\gamma+1)g^{(3)}}{4G^3} + \frac{103+23\gamma}{60G}\right)R_0^{-1/4} - \left(\frac{(\gamma+1)g^{(3)}}{4G^3} + \frac{2(\gamma+1)\ln R_0}{5G} - \right.$$
$$\left.\left.\frac{4(\gamma+1)}{5G}\right)R_0^{1/4}\right](t+R_0)^{3/4} + \frac{433R_0^{-3/4}(t+R_0)^{-5/4}}{23040G^3} +$$
$$\left(\frac{11R_0^{7/4}}{256G^3} + \frac{11(\gamma+1)R_0^{1/4}}{16G} - \frac{R_0^{-3/4}}{8G^3}\right)(t+R_0)^{-1/4} -$$
$$\frac{(103+23\gamma)R_0^{-1/4}(t+R_0)^{3/4}}{60G} - \left(\frac{R_0^{-7/4}}{40G^3} + \frac{2(\gamma+1)R_0^{1/4}}{5G}\right)(t+R_0)^{3/4} \times$$
$$\ln(t+R_0) + \frac{4(\gamma+1)R_0^{-1/4}(t+R_0)^{5/4}}{5G}, \qquad G = [g^{(2)}]^{1/2}$$

and restrict ourselves to an approximate expression for the function $\Psi'(r, t)$ in the form

$$\Psi'(r, t) = a_0 + a_1 r + b^{(1)}r^{3/2} + b^{(2)}r^2 b^{(3)}r^{5/2} \tag{3.2}$$

under the assumption that within the region of perturbed motion, the velocities ahead of the weak discontinuity L are small compared with the speed of sound. From (3.1) we obtain

$$\xi = a_1 + {}^3/_2 b^{(1)}r^{1/2} + 2b^{(2)}r + {}^5/_2 b^{(3)}r^{3/2} \tag{3.3}$$

The instant of disruption of the potential flow corresponds to the smallest value of t^* at which $\partial r / \partial \zeta$ becomes infinite or $\partial \zeta / \partial r$ becomes zero [4]. The place and time of the "gradient collapse" is determined by solving simultaneously the following two equations

$$\left(\frac{\partial \xi}{\partial r}\right)_t = 0, \qquad \left(\frac{\partial^2 \xi}{\partial r^2}\right)_t = 0 \tag{3.4}$$

where ξ is taken from (3.3). The general case of disruption of potential flows directly on the weak discontinuity L_t was studied in [1]. In the class of the flows in question the derivatives of the gasdynamic quantities are zero at the surface L_t at all times, and derivatives of infinite value may appear either within the region of compression contained between the surfaces S_t and L_t, or directly at the surface S_t. No analytic solution of the system (3.4) could be obtained for $\Psi'(r, t)$ from (3.2), but the time t^* and position ξ^* of the onset of disruption of the potential flow can always be found by numerical methods.

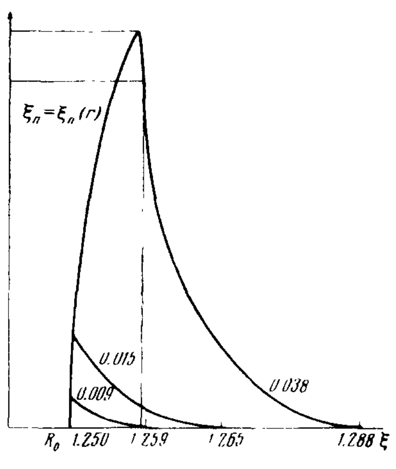

Fig. 1

Example. Let the gas at the initial instant $t = 0$ be at rest outside a cylindrical piston of radius R_0. At the instant $t = 0$ the piston begins to expand according to the law

$$V_n = g^{(2)}t^2 + g^{(3)}t^3 + g^{(4)}t^4$$

Figure 1 gives for the above law of motion the velocity profiles at three instants of time. The profiles show that a shock wave is formed at some point between the piston surface S_t and the surface of weak discontinuity L_t. Numerical computations for $\gamma = 1.4$, $g^{(2)} = 0.25$, $g^{(3)} = 2$, $g^{(4)} = 0.2$ and $R_0 = 1{,}25$ gave

$$t^* = 0.038, \quad \zeta^* = 1.259$$

Note. Just as in [2], we can use the expression (3.3) for Ψ to describe the flows behind the shock wave appearing at $t = t^*$, under the assumption that the wave remains weak.

4. Let us investigate the convergence of the functional series (1.5) for the case of flows with cylindrical symmetry. We pass from r to a new variable z according to the formula

$$\sqrt{r} = z \tag{4.1}$$

Then (1.1) becomes

$$\frac{\Psi_{tt}\Psi_z\Psi_{zz}}{8z^5} - \frac{\Psi_{tt}\Psi_z^2}{8z^6} + \frac{1}{\varkappa}\left[\frac{\Psi_t\Psi_{zz}}{4z^2} + \frac{\Psi_t\Psi_z}{4z^3} - \frac{z^2\Psi_{zz}}{8} - \frac{z\Psi_z}{8}\right] - \frac{\Psi_{zt}^2\Psi_z}{8z^5} + \frac{\Psi_{zt}\Psi_z}{2z^2} - \frac{z\Psi_z}{2} = 0 \tag{4.2}$$

The boundary conditions at $z = 0$ are

$$\Psi(0, t) = \varkappa t + \text{const}, \quad \Psi_z(0, t) = 0, \quad \Psi_{zz}(0, t) = 2(t + R_0) \tag{4.3}$$

Let $\chi(t)$ be the specified law of motion of the piston

$$\chi(0) = R_0, \qquad \chi'(0) = 0, \qquad \chi''(0) = 0, \quad \chi'''(0) \neq 0 \tag{4.4}$$

From (3.1) follows

$$\chi'(t) = \Psi_{rr}(\chi'(t), t)\,\chi''(t) + \Psi_{rt}(\chi'(t), t) \tag{4.5}$$

From (4.5), using the relation $\chi'(t) = r(t) = z^2(t)$ at the piston, we obtain

$$z^2 = \frac{z\Psi_{zz} - \Psi_z}{2z^2}\, z' + \frac{\Psi_{zt}}{2z} \tag{4.6}$$

From the conditions (4.4) it follows that the function $z = \sqrt{r(t)} = \sqrt{\chi'(t)}$ is analytic in the vicinity of zero and $dz/dt|_{t=0} \neq 0$. Then for small z there exists an inverse function and the relation (4.6) can be written in the form

$$z^2 = \frac{z\Psi_{zz} - \Psi_z}{2z^2}\,\frac{1}{\eta'(z)} + \frac{\Psi_{zt}}{2z} \quad \text{for } t = \eta(z) \tag{4.7}$$

Thus, the substitution (4.1) yields (4.2) with the conditions (4.3) and (4.7). Let us now replace the function $\Psi(z, t)$ by another unknown function $\Phi(z, t)$ according to the formula

$$\Psi(z, t) = \varkappa t + \text{const} + (t + R_0)\, z^2 + \Phi(z, t)\, z^3 \tag{4.8}$$

The equation for $\Phi(z, t)$ which follows from (4.2) and the boundary condition (4.7) now become

$$A\Phi_t + zB\Phi_{zt} = z^2 f_1(t, z, \Phi, \Phi_z, \Phi_t, \Phi_{tt}, \Phi_{zt}, \Phi_{zz}) + zf_2(z, t, \Phi, \Phi_z, \Phi_{tt}) + f_3(z, t, \Phi) \quad (4.9)$$

$$\Phi + Cz\Phi_z + z^2\Phi_{zz} = f_4(z, t, z\Phi_t, z^2\Phi_{zt}) \quad \text{for} \quad t = \eta(z)$$

where A, B and C are positive constants and the functions f_1, f_2, f_3 and f_4 are analytic in all variables. By the theorem proved in [5] the problem (4.9) has a unique analytic solution and the series (1.5) as well as the series for the second order derivatives obtained from (1.5) converge in some neighborhood of the point (0, 0).

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ON FORCES DUE TO BARNETT STRESSES ACTING ON BODIES IN A GAS

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Proof is given of certain statements about forces acting on uniformly heated bodies in a gas. It is shown that bodies heated to different temperatures repel each other, while a heated and a cooled body are mutually attracted. A new form of thermophoresis is indicated. These phenomena are the result of Barnett thermal stresses. The existence of similar effects induced by concentration stresses in gas mixtures is established.

1. Fundamental relationships. When defining slow (characteristic Reynolds number $R \approx 1$ and Mach number $M \ll 1$) flows of gas in a substantially nonuniform temperature field, i.e. whose characteristic relative temperature differentials $\tau_* \approx 1$, it is necessary to take into consideration Barnett thermal stresses